Mode-Matching Approach for Superconducting Planar Transmission Lines Including Finite Conductor Thickness

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Abstract—**An analysis of superconducting planar transmission lines is presented. The full-wave mode-matching approach holds for arbitrary values of conductor thickness and does not involve approximations such as the skin-effect description or the resistive boundary condition. Results on phase constant and attenuation of a superconducting CPW are reported including both conductor-thickness dependence and frequency characteristics. The results demonstrate that the high-T_c superconductor materials available to date greatly improve the performance of coplanar waveguides as used in mm-wave MMIC's.**

I. Introduction

The recent developments in the area of high-T_c superconductor materials give rise to a variety of applications in microwave and mm-wave circuit elements. Their low-loss features make them suitable particularly for miniaturized structures such as CPW transmission lines and resonators (see e.g., [1]). In this situation, reliable modeling tools are to be developed enabling one to predict the potential advantages of superconducting materials for a specific device.

Regarding transmission-line analysis, several theoretical approaches are reported in the literature (e.g., [1]–[4]). Most of them require assumptions on the ratio between conductor thickness t and superconductor penetration depth λ_s. The common perturbation technique holds if λ_s is small compared to the thickness t, the resistive boundary condition [2], [4] applies to the opposite case. Regarding realistic CPW structures (e.g., [1]), however, one finds that none of these conditions is fulfilled since conductor thickness and penetration depth are of the same order of magnitude.

Therefore, the mode-matching approach presented in this paper uses a more general description for the superconductors that holds also in the intermediate range, which is of special interest here.

II. Method of Analysis and Convergence Behavior

It is generally accepted that the coherence length of the high-T_c materials remains small compared to the superconductor penetration depth λ_s and thus the current-field behavior of the superconductors is local. Consequently, the phenomenological two-fluid model can be applied characterizing the superconducting regions by a complex-valued conductivity. As well known, one obtains for the resulting complex permittivity

$$\epsilon = \epsilon_r \epsilon_0 - \frac{1}{\omega^2 \mu \lambda_s^2} - j\frac{\kappa}{\omega}. \tag{1}$$

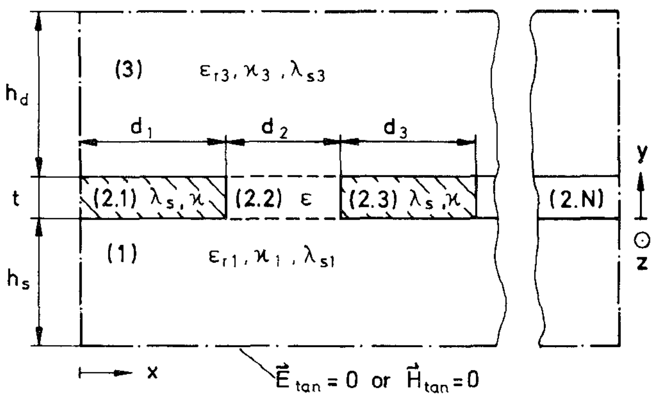

Fig. 1. Waveguide model (all subregions are characterized by their complex dielectric constant).

Formally, this does not differ from the case of normal conductivity. We developed, therefore, an extension of the mode-matching method employed already in [5] for the analysis of planar transmission lines with normal conductors.

Fig. 1 shows the model used. Each subregion is described by its complex permittivity. Thus the losses and superconducting properties are included generally without further limitations. The lower and the upper medium, which are homogeneous with respect to the x coordinate, can be substituted by a layered structure.

From the formal similarities previously mentioned, one would expect a convergence behavior resembling that of the normal conducting case. We found, however, some essential differences. Generally speaking, they can be attributed to the fact that, though both normal conductivity and superconductivity result in a field displacement towards the conductor surface, the inherent mechanisms are not the same. In normal conducting media, this effect is produced by losses, whereas superconductivity, on principle, represents a lossless mechanism.

Fig. 2 illustrates the resulting behavior of the longitudinal-section H_x eigenfunctions for the superconducting case (details of the method of analysis can be found in [5]). In order to demonstrate the basic effects, a simple microstrip geometry is chosen (see caption of Fig. 2). The real parts of

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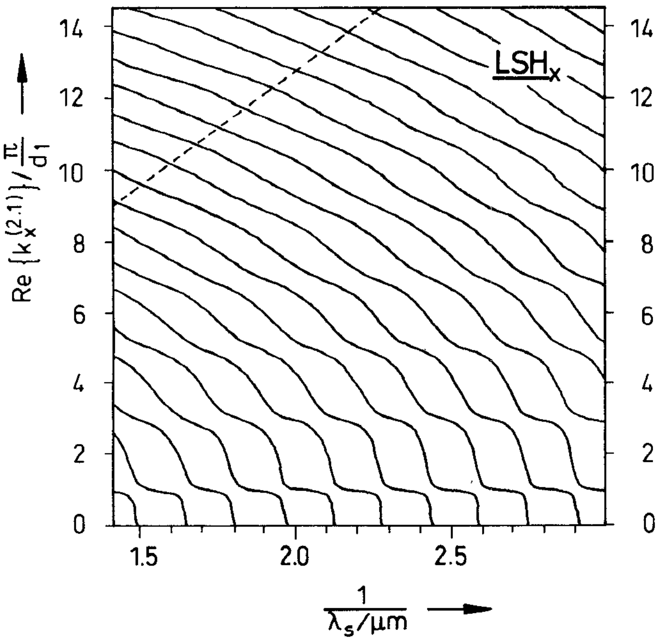

Fig. 2. Real part of the normalized separation constants k_{xm} of the LSH_x longitudinal-section eigenfunctions in region (2.1) against the inverse penetration depth $1/\lambda_s$. The dashed line refers to Real$\{k_x\} = \lambda_s^{-1}$. A microstrip geometry is considered at 1 GHz. Region (2.1) in Fig. 1 consists of a lossless dielectric (d_1 = 20 μm) and region (2.2) of a superconducting strip (d_2 = 10 μm) with $\kappa = 10^5$ S/m.

the LSH_x separation constants k_{xm} are plotted as a function of the inverse superconductor penetration depth $1/\lambda_s$. The dashed line denotes the curve Real$\{k_x\} = \lambda_s^{-1}$.

What is important to note is that the k_{xm} do not approximate constant values when decreasing the penetration depth λ_s (i.e., increasing $1/\lambda_s$). For Real$\{k_{xm}\} \le \lambda_s^{-1}$, one observes resonance-like k_x transitions indicating that strong coupling between the different eigenfunctions occurs.

As a consequence, also the convergence behavior of the complex propagation constant $k_z = \beta - j\alpha$ deviates from the smooth characteristic found in the normal conducting case. Problems arise if one takes into account in the layered media ($2.1 \cdots 2.N$) only a part of the LSH_x eigenfunctions within the range Real$\{k_{xm}\} \le \lambda_s^{-1}$. This leads to a nonuniform behavior with increasing number of eigenfunctions, which often does not allow to extrapolate the limiting value with sufficient accuracy. On the other hand, when including the *complete* range Real$\{k_{xm}\} \le \lambda_s^{-1}$, a smooth convergence is found likewise the normal conducting case.

This means that the description of superconductors requires a minimum number of eigenfunctions to be included. This number depends on the ratio between the total width in x direction and the penetration depth λ_s and determines the matrix size of the resulting k_z-eigenvalue problem. Regarding miniaturized transmission lines as found in MMIC structures such a restriction does not cause difficulties (e.g., for the superconducting CPW structure of Fig. 4, 144 LS_x eigenfunctions in the conductor plane and 151 eigenfunctions in the regions homogeneous along x are applied). For lateral line geometries wider than about $1000 \cdot \lambda_s$, however, one faces severe numerical efforts which, in practice, limit the applicability of our method.

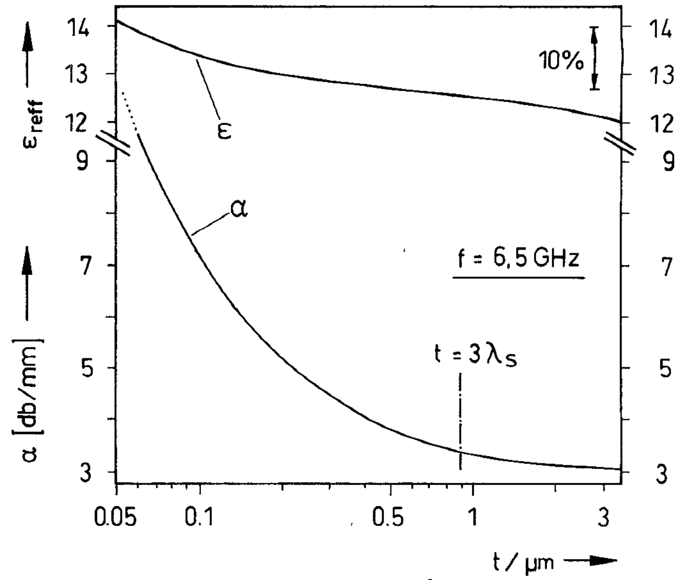

Fig. 3. Effective permittivity $\epsilon_{r\,\mathrm{eff}} = (\beta/\beta_0)^2$ and attenuation α against conductor thickness t. Superconducting CPW according to [1]: d_1 = 170 μm, slot width d_2 = 46 μm, center conductor width $2d_3$ = 74 μm, conductor thickness t = 0.28 μm; 500 μm thick $LaAlO_3$ substrate with ϵ_r = 24, $\kappa = 1.1 \cdot 10^{-3}$ S/m; YBaCuO superconductor: λ_s = 0.3 μm, $\kappa = 8.2 \cdot 10^5$ S/m at 77 K.

In order to check validity, the results were compared with analytical parallel-plate expressions that can be derived for the wide superconducting microstrip (see [2], (32) and (34)). We found excellent agreement (less than 0.5% deviation). Also, the CPW resonators described in [1] were calculated. We obtain quality factors $\beta/(2\alpha)$ of 2110 (resonator 1) and 4640 (resonator 2), respectively. Although these values are higher than the measured ones the fit is better than that for the perturbation approach given in [1]. Evaluating the deviations one should bear in mind also that the quality factor of such resonators depends strongly on the superconductor parameters (λ_s and the normal conductivity κ), which are difficult to measure accurately.

III. Results

Fig. 3 shows the effective dielectric constant $\epsilon_{r\,\mathrm{eff}} = (\beta/\beta_0)^2$ and the attenuation as a function of conductor thickness t. A CPW with superconducting films according to [1] (resonator No. 2) is studied. Regarding α, one finds a steep increase for thickness values smaller than $3\lambda_s$, which corresponds to the case of normal conductivity [5]. Also, $\epsilon_{r\,\mathrm{eff}}$ exhibits a significant t dependence. For low values of t, this effect can be attributed to the internal inductance, whereas for growing thicknesses the field distribution within the slot area plays a major role.

The miniaturized CPW geometries as used in conventional monolithic mm-wave integrated circuits (MMIC's) show relatively high conductor loss. It is, therefore, interesting to evaluate their propagation behavior when replacing the conductors by high-T_c materials. Fig. 4 presents a comparison in terms of the frequency dependence for 0.3 μm conductor thickness ($LaAlO_3$ substrate). One finds a loss reduction due to the superconducting properties by a factor of more than 10. Together with the small line dimensions, this also leads to an ideal low-dispersive behavior in $\epsilon_{r\,\mathrm{eff}}$ and the character-

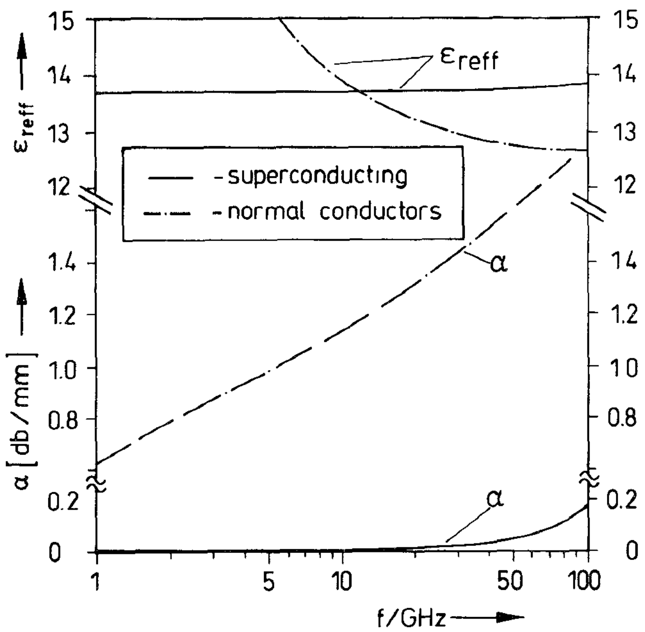

Fig. 4. Effective permittivity and attenuation as a function of frequency. Comparison between superconducting CPW (solid lines) ($\lambda_s = 0.3$ μm, $\kappa = 8.2 \cdot 10^5$ S/m at 77 K—see [1]) and metallic conductors (dashed lines) with conductivity $\kappa = 3 \cdot 10^7$ S/m. Line geometry (see Fig. 1): $d_1 = 50$ μm, slot width $d_2 = 10$ μm, center conductor width $2d_3 = 15$ μm, conductor thickness $t = 0.3$ μm, $h_s = 600$ μm, $LaAlO_3$ substrate with $\epsilon_r = 24$ and $\kappa = 1.1 \cdot 10^{-3}$ S/m.

istic impedance. In the superconducting case, the attenuation follows roughly a quadratic law as expected.

IV. Conclusion

The mode-matching approach represents an accurate description of lossy planar transmission lines of arbitrary conductor thickness, also in the presence of local superconductors. In contrast to the case of normal conductors, however, its applicability becomes problematic for large ratios between lateral width and penetration depth due to severe numerical efforts.

A considerable improvement of the propagation characteristics is achieved when substituting the metallic layers of mm-wave coplanar waveguides by state-of-the-art high-T_c superconductors. This comprises a reduction in α of more than one order of magnitude as well as an extremely low dispersion up to 100 GHz.

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